On the Scattering of Homogeneous β-Rays and the Number of Electrons in the Atom.

By J. A. Crowther, M.A., Fellow of St. John's College, Cambridge.

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Introduction.

The subject of the transmission of β-rays through matter has, from time to time, received considerable attention. Apart from any intrinsic interest the problems involved are of considerable theoretical importance. Owing to the high velocity of the β-rays, the collisions, to which the absorption of the rays must be ascribed, take place not with the atom as a whole, but with its constituent parts, and thus from a study of the behaviour of the rays during their passage through matter we may hope to gain considerable information as to the constitution of the atom.

Until very recently it was thought that the phenomena involved in the absorption of the β-rays were very simple. It was early shown that the β-rays from a single radioactive substance, such as uranium X for example, were absorbed by light substances, such as aluminium, according to an exponential law. For the heavier elements, such as tin or platinum, the absorption curve at first descended rather more steeply than the true exponential, but finally became exponential after the rays had passed through some small thickness of the absorbing material. This law has been tested for a large number of elements and compounds by the present writer,* and with a high degree of accuracy, for a few substances, by N. R. Campbell.†

This being so, the fact that a given bundle of rays was absorbed according to an exponential law was regarded as proving that the rays were homogeneous, and, further, that their velocity did not appreciably alter during their passage through the absorbing material. Quite recently, however, W. Wilson‡ published results which threw some doubt on both these conclusions. Instead of working with the β-rays from a single radioactive substance, he used the mixed bundle of rays from radium, employing a magnetic field to sort them into a series of nearly homogeneous pencils. He then found that the absorption of the "homogeneous" rays thus obtained did not follow an exponential law, but that it could best be represented by

* J. A. Crowther, 'Phil. Mag.,' 1906, vol. 12, p. 379.
† N. R. Campbell, 'Phil. Mag.,' 1909, vol. 17, p. 180.
‡ W. Wilson, 'Roy. Soc. Proc.,' 1909, A, vol. 82, p. 612.

a linear relation. As we shall see later, this linear relationship is not strictly true, the absorption curve for a substance such as aluminium having in reality two points of inflection. We shall return to this point later.

In the same paper Wilson also gave results suggesting that the β-rays lose velocity in their passage through the absorbing medium. This point has been since investigated by the present writer,* who found, by direct measurements of the velocity of the rays before and after transmission through absorbing sheets, that a small but distinct loss of velocity did occur, and similar results have been recorded still more recently by Wilson† himself.

In a previous paper‡ I have shown that the absorption of a parallel pencil of β-rays may conveniently be divided into two stages: firstly, the scattering or diffusion of the pencil, and, secondly, the more gradual absorption of the diffuse rays. Without considering for the moment whether the two phenomena are really distinct, or whether they may not be merely two different expressions of the same phenomenon, we may regard it as an experimental fact that a parallel pencil of β-rays is considerably scattered in a thickness of material too small to produce any appreciable absorption in a uniformly diffused beam. It will be convenient, therefore, to consider the two phenomena separately.

Moreover, as now appears, the measurement of the absorption of the rays, as made by ionisation methods, is open to some uncertainty. In all ionisation methods it is assumed that the amount of ionisation produced is a simple measure of the intensity of the rays producing it. So long as the velocity of the rays remains the same, this is in all probability accurately the case. When, however, the velocity is changing, as appears to be the case to some extent during the absorption of homogeneous β-rays, the assumption is no longer so obvious, and it is uncertain whether the ionisation produced measures the energy of the rays, their number, or some function of the two; while if secondary radiation is at all important it may also depend upon the substance and shape of the ionisation chamber.

Measurements on the scattering of a pencil of β-rays do not suffer from these uncertainties. As pointed out above, the rays are scattered in a thickness of material far too small to produce any appreciable effect on the velocity of the rays, and possible errors due to this cause are therefore eliminated. For this reason experiments on the scattering of the β-rays are

* J. A. Crowther, 'Camb. Phil. Soc. Proc.,' 1910, vol. 15, pt. 5, p. 442.

† W. Wilson, 'Roy. Soc. Proc.,' June, 1910.

‡ J. A. Crowther, 'Roy. Soc. Proc.,' 1908, A, vol. 80, p. 186.

far more suitable for the investigation of the constitution of the atom than the more complex phenomena of absorption.

Theory of the Scattering.

Prof. Sir J. J. Thomson* has very recently published a theoretical solution of the problem of the scattering of rapidly moving electrified particles. For the complete investigation the original memoir should be consulted. We shall, however, abstract here as much as will be necessary for the purposes of the present paper.

When a β-particle passes through an atom of a substance it will be deflected. The amount of this deflection will, of course, depend upon the way the particle strikes the atom. There will, however, be a mean value for this deflection, and, considering only the effects produced by large numbers of the β-particles, it may be assumed that each particle suffers the mean deflection. Since the direction of the deflection is arbitrary, the problem is the same as that of finding the average value of the resultant of n displacements of arbitrary phase and of constant amplitude θ. This value is known to be $\sqrt{n}.\theta$. Thus if N is the number of atoms per unit volume, b is the radius of an atom, and θ the mean deflection of a β-particle produced by an atom, then the mean value of the deflection experienced by a particle in passing through a plate of thickness t is $\sqrt{(\mathrm{N}\pi b^2 t)}.\theta$. Calling this angle ϕ_m, we have for any given substance

$$\phi_m/\sqrt{t} = \text{constant}.$$

If ϕ is a given angle, the probability that the deflection is greater than ϕ is equal to $e^{-\phi^2/n\theta^2}$ or $e^{-\phi^2/ct\theta^2}$, where c is a constant, since n varies directly as the thickness t. Thus, if t_m is the thickness for which this probability is one-half, we have

$$e^{-\phi^2/ct\theta^2} = \tfrac{1}{2}, \tag{a}$$

or

$$\phi/\sqrt{t_m} = \theta\sqrt{(c\log_e 2)}, \tag{1}$$

which is constant for any given absorbing substance.

The probability that the deflection is less than ϕ is equal to $(1-e^{-\phi^2/ct\theta^2})$, that is to

$$1-e^{-k/t} \tag{2}$$

where k is a constant for any given value of ϕ.

Both these relationships lend themselves readily to experimental verification.

To calculate the value of θ, the atom is regarded as consisting of N_0

* Prof. Sir J. J. Thomson, 'Camb. Phil. Soc. Proc.,' 1910, vol. 15, Pt. 5.

negative corpuscles accompanied by an equal quantity of positive electrification. The deflection of the β-particle will thus arise from two causes:—

(1) The repulsion of the negative corpuscles distributed through the atom; and

(2) The attraction of the positive electrification.

The deflection due to the latter will depend upon whether the positive electricity is uniformly distributed through the atom, or whether it is divided up into small units. Prof. Thomson considers each of these suppositions separately. It will be sufficient here to give the results which he arrives at. He found that the mean deflection ϕ_m produced by passage through a thin plate of thickness t was given by

$$\phi_m = \frac{e^2}{mv^2}\left\{\frac{384}{25}N_0+\frac{\pi^2}{16}N_0^2\right\}^{\frac{1}{2}}\sqrt{N\pi t} \qquad (A)$$

if the positive electricity is uniformly distributed; and by

$$\phi_m = \frac{e^2}{mv^2}\left[\frac{384}{25}N_0\left\{2-\left(1-\frac{\pi}{8}\right)\sigma^{\frac{1}{3}}\right\}\right]^{\frac{1}{2}}\sqrt{N\pi t} \qquad (B)$$

if the positive electricity is in small separate units; σ being here the ratio of the volume occupied by the positive electricity to the volume of the atom.

The main object of the present experiments has been to test as completely as possible the agreement of the above relationships with the experimental facts, and, if the agreement proved to be satisfactory, to determine the value of N_0, the number of corpuscles in an atom, for a number of different elements.

Experimental Details.

Before proceeding to describe the results obtained, it will be necessary first to give a brief account of the experimental methods employed.

The source of the β-radiation was a sample of about 30 milligrams of radium bromide, in radioactive equilibrium, enclosed in a small glass tube. This was found to give out β-rays having a continuous range of velocities from about $2{\cdot}35\times10^{10}$ to $2{\cdot}92\times10^{10}$ cm. per second. These were sorted into bundles of rays having nearly the same velocity by a magnetic field.

It may be permissible here to point out the limitations of the magnetic deflection method, especially as they do not seem to have been quite grasped by some previous users of the method. Various screens are arranged, as in fig. 1, to mark out a circular path ACB, and a magnetic field is applied at right angles to the plane of the paper. By suitably adjusting the strength of this field, rays entering the system at A can be made to trace out the path ACB and emerge through B into a suitable measuring vessel. If, however, the apertures have a finite size, as must always be the case in practice,

the path ACB is not the only possible path for rays to follow in order to get through the aperture B. A little consideration will show that rays of uniform velocity, if entering the field obliquely, may be transmitted through the system along paths such as *acb* or *a'c'b'*, even when the field is too strong or too weak to deflect the normal rays along the path ACB. In this way, even if the rays are uniform to begin with, there will be a definite range of field strength for which some of the rays can pass through the system. Conversely, if the rays are not homogeneous to begin with—if, for example, we are dealing with the whole of the β-rays from radium—there will be for every magnetic field a finite range of velocities which the β-rays may possess and still be able to pass through the system. Unless the size of the apertures employed is quite small compared with the radius of the path, this range of velocities will be very considerable, and the emergent beam

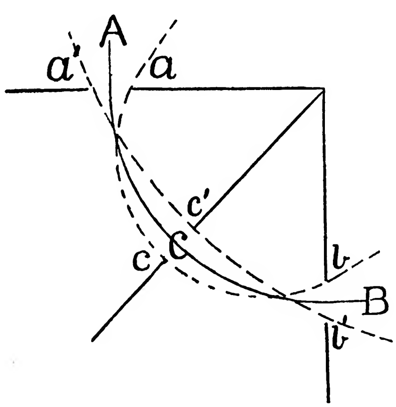

Fig. 1.

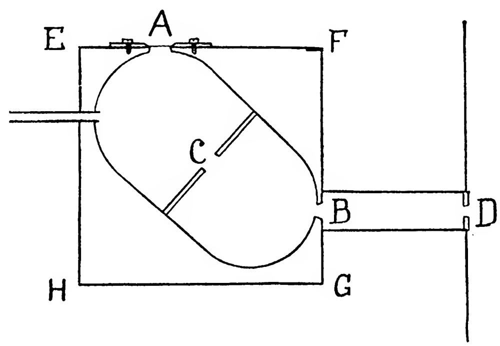

Fig. 2.

far from homogeneous. It is important, therefore, that the apertures be kept as small as possible. On the other hand, as the quantity of β-rays which can be radiated through a given area by any radioactive substance is limited, the size of the apertures is ultimately fixed by the sensitiveness of the measuring arrangements.

One further point of importance remains to be noticed, namely, the possible scattering of the rays during their passage through the air. Some preliminary experiments* which were undertaken to test this matter showed that if the path of the rays was at all long, say, even 4 or 5 cm., the scattering of the rays by the air was very considerable. In order to ensure the homogeneity of the transmitted rays, and still more in order to obtain a parallel pencil such as was required for the present experiments, it was necessary to

* J. A. Crowther, 'Camb. Phil. Soc. Proc.,' 1909, vol. 15, pt. 3, p. 273.

perform the whole operation of deflecting the rays, and forming them into a suitable pencil, *in vacuo.*

The form of apparatus finally decided upon is shown in section in fig. 2, The portion EFGH is placed between the poles of an electromagnet, so that the edges of the pole pieces lie along EF and FG. The tube BD projects from the magnetic field and is screened from it as carefully as possible by a thick block of soft iron. The radius of curvature of the path of the rays is 4 cm., the diameter of the apertures at A and C is 0·5 cm., and the diameter of those at B and D 0·3 cm. The depth of the box at right angles to the plane of the paper is 1·8 cm. The different apertures are bevelled and the screens and sides of the box lined with aluminium to avoid as far as possible any secondary radiation. The radium was placed immediately above A, and the aperture at A was closed with thin aluminium foil (0·002 cm.), while the tube D was soldered into the scattering chamber The whole could be exhausted through the tube t, which was connected to a pressure gauge and pump.

With this apparatus a very satisfactory pencil of homogeneous β-rays was produced. By constructing a pair of such chambers and using the second to measure the velocity of the rays transmitted through the first, it was found that the extreme velocities of the β-rays passing through either of the systems did not differ by as much as 1 per cent. from the mean. The greatest possible deviation from the normal of the rays passing through D was less than 5°.

The emergent pencil of rays after leaving D entered the exhausted chamber S (fig. 3). By means of a rotating sector P, which could be operated from without, the rays could be allowed to go straight on, in which case the whole of the pencil passed into the ionisation chamber, or made to pass through different absorbing screens mounted upon the sector. The opposite face of the chamber S was closed with a thin aluminium window 6 cm. in diameter, but stops of varying aperture could be inserted in the window at R, to limit the emergent beam to any desired angle.

The ionisation chamber T calls for little comment. The face towards the chamber S was closed with aluminium leaf, and an aluminium leaf mounted on a wire ring also formed the inner electrode e. The other face was hollowed out, so that at whatever angle the rays emerged from the screens at P they should all have the same length of path in the ionisation chamber itself. In this way any correction for the increased ionisation due to the increasing length of path of the more oblique rays was rendered unnecessary.

The inner electrode e was connected by a wire passing through an earthed

tube to the key K and the inclined electroscope W. On account of the large ionisation produced in the neighbourhood of the apparatus by the γ-rays from the radium, the connecting tubes, etc., were filled with sulphur, and where this was impossible, as in the key, the actual air space was kept as small as possible.

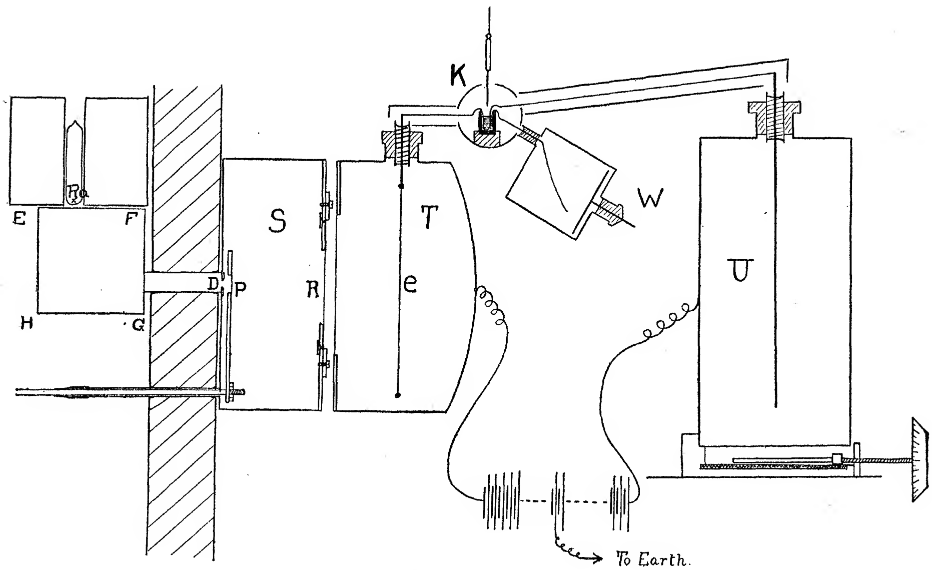

Fig. 3.

The actual measurements were made by a compensating method, the uranium shutter compensator devised for the measurement of the scattering of the uranium rays being again employed. As this apparatus is fully described in a previous paper,* it will not be necessary to give the details here. In brief the method consists in attaching to the same electroscope as the ionisation chamber S a second ionisation chamber U charged to the opposite potential. The ionisation in the latter can be varied in a known manner by means of a shutter sliding over a surface of uranium oxide. The shutter is moved in or out until a balance is obtained between the two ionisation currents, when the ionisation can be obtained from the reading of the shutter.

This method has two special advantages in the present experiments. As a little consideration will show, the processes employed to obtain a satisfactory pencil of homogeneous rays cut down the β-radiation which eventually reaches the ionisation chamber T to a very minute fraction of its original

* J. A. Crowther, 'Roy. Soc. Proc.,' 1908, A, vol. 80, p. 186.

amount. The γ-rays, on the other hand, though shielded off as far as possible by 4 cm. of lead and 3 cm. of iron, still produced a very considerable ionisation in the chamber T. If a direct timing method had been employed, the ionisation due to the β-rays would have come out as a relatively small difference between two large ionisation currents. As the percentage accuracy with which an ionisation current can be measured does not increase (beyond a certain point) with the magnitude of the current, this difference could not have been measured with any degree of accuracy. With the compensating method, however, the ionisation due to the γ-rays can be compensated to begin with, and the actual ionisation due to the β-rays themselves can be measured with precisely the same degree of accuracy as if the γ-ray ionisation were not present.

The second advantage is that the results are not affected by defects in the insulation, as would be the case if a timing method were employed. As defective insulation only comes into play when the gold leaf system has actually acquired a charge, it cannot affect the direction in which the leaf would begin to move, and hence cannot affect the final balance. In a timing method of measurement defects in the insulation will alter the rate of movement of the leaf, and may cause considerable errors in the results. This is of importance in the present case, as it is always difficult to secure a high degree of insulation in the presence of any considerable amount of radium.

A few words may be added to indicate the order of accuracy to be expected in the measurements. The maximum ionisation I_0 due to the β-radiation amounted to about 30 on the compensator scale, and it was possible to balance to within 0·5 of a division. The readings could therefore be made to an order of accuracy of about 2 per cent. of the maximum ionisation. After the rays had been cut down to one half (as in the measurement of t_m, the thickness required to produce this effect), readings could therefore be made to an accuracy of about 4 per cent., and results obtained on different occasions were generally found to agree to within about this amount. The probable error in the ionisation measurements may therefore be regarded as being about 4 or 5 per cent.

The angle of emergence of the rays was obtained by measuring the diameter of the stop and the fixed distance of the stop from the scattering layer; these could readily be ascertained to 1 per cent. The thickness of the scattering layers was measured by finding the weight of sheets of known area. This operation could also be readily performed to an accuracy of 1 per cent. If, however, the sheet used was not of uniform thickness, an error might be introduced from the fact that the rays actually passed

through only a small part of the area. Any error due to this cause can be eliminated by taking a number of readings, using different portions of the sheet, and any serious departure from uniformity was only to be expected in the case of very thin leaf.

Experimental Results.

The theory which has been described in the previous pages lends itself to experimental tests in three different ways.

(i) If the stop at R remains the same, so that the angle ϕ over which the rays can emerge into the ionisation chamber remains constant while the thickness t of the absorbing screen is varied, we have from equation (2)

$$I/I_0 = 1 - e^{-k/t},$$

where k is a constant when ϕ is constant. This may be conveniently tested in the form

$$t \log_e (1 - I/I_0) = \text{constant}. \tag{3}$$

(ii) If t_m is the thickness of material required to cut down the radiation through a given stop of angle ϕ to one half its original value, we have, from equation (1),

$$\phi / \sqrt{t_m} = \text{constant}. \tag{4}$$

(iii) From either of the equations (A) and (B) we have

$$\frac{\phi}{\sqrt{t_m}} \propto \frac{1}{mv^2}$$

when the absorbing medium remains the same and the velocity of the incident rays is varied. Keeping the same stop, and therefore keeping ϕ constant, we have

$$mv^2 / \sqrt{t_m} = \text{constant}. \tag{5}$$

Using aluminium as the absorbing medium, a series of experiments were made to test all these relationships. A considerable number of experiments were also made, using platinum as the absorbing substance, this element having been selected as being as unlike aluminium, both in density, atomic weight, and the possibility of a secondary radiation, as possible. In the case of every substance employed, sufficient readings were made to show that the relationship (4), which is the real criterion of the applicability of the theory, was valid. The results obtained are contained in the following tables.

Table I is a specimen set of readings for aluminium, obtained by keeping the stop which limits the angle of emergence of the rays fixed, and interposing varying thicknesses of aluminium in the path of the rays. I_0 is the intensity of the incident beam; I the intensity of the rays getting through the stop after passing through a thickness t of aluminium. Thus I/I_0

measures the proportion of the β-rays which have their path deflected through an angle less than ϕ in a thickness of aluminium t. The values of I/I_0 plotted against the corresponding thickness t are given in fig. 4.

Table I.—Aluminium.

$\phi = 18°$. $v = 2{\cdot}64 \times 10^{10}$ cm./sec.

t.	I/I_0.	$1-I/I_0$.	$\log_e(1-I/I_0)$.	$t\log_e(1-I/I_0)$.
cms.				
0·00069	0·98	0·02	−3·9	−0·0027
0·00117	0·92	0·08	−2·5	−0·0029
0·00225	0·72	0·28	−1·27	−0·0028
0·00450	0·48	0·52	−0·64	−0·0029
0·00675	0·36	0·64	−0·43	−0·0029

Hence $(1-I/I_0)$ is the proportion of the rays which have their path deflected through an angle greater than ϕ. As shown above (equation 3), $t\log_e(1-I/I_0)$ should be a constant for a given angle ϕ.

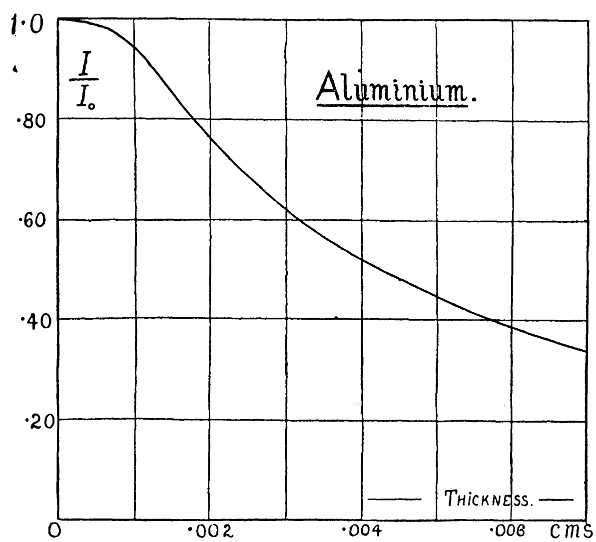

Fig. 4.

The last column of Table I gives the values of this quantity for an angular aperture $\phi = 18°$, for varying thicknesses of aluminium. It will be seen that the numbers obtained are constant to well within the probable errors of the experiment. Similar experiments were made with other stops, and also with platinum as the absorbing substance. As the results were precisely similar, it will not be necessary to reproduce them at length.

Table II.—Aluminium.

$v = 2{\cdot}681 \times 10^{10}$ cm./sec.

ϕ (radians).	t_m (cms.).	$\phi/\sqrt{t_m}$.
0·20	0·0022	4·3
0·24	0·0033	4·2
0·31	0·0054	4·2
0·41	0·0090	4·3
1·20	0·070	4·5

Table II gives the thickness t_m of aluminium which is necessary to reduce the β-radiation through a given stop of angular aperture ϕ to one-half its original value. The last column gives the values of $\phi/\sqrt{t_m}$, which according to the theory should be constant. The first four readings were obtained with the apparatus described above. The last comes from some experiments to be described later on the absorption of the β-rays, and is placed here for purposes of comparison. It will be seen that the theory is again borne out by the experimental results.

Table III.—Aluminium.

$\phi = 18^\circ$.

HR.	v.	mv^2/e.	t_m (cms.).	$mv^2/e\sqrt{t_m}$.
2260	$2{\cdot}40 \times 10^{10}$	$5{\cdot}41 \times 10^{13}$	0·00183	$1{\cdot}26 \times 10^{15}$
2870	$2{\cdot}58 \times 10^{10}$	$7{\cdot}40 \times 10^{13}$	0·00406	$1{\cdot}16 \times 10^{15}$
3420	$2{\cdot}68 \times 10^{10}$	$9{\cdot}15 \times 10^{13}$	0·0056	$1{\cdot}23 \times 10^{15}$
4050	$2{\cdot}77 \times 10^{10}$	$11{\cdot}2 \times 10^{13}$	0·0084	$1{\cdot}22 \times 10^{15}$
4840	$2{\cdot}83 \times 10^{10}$	$13{\cdot}7 \times 10^{13}$	0·0122	$1{\cdot}25 \times 10^{15}$
6500	$2{\cdot}90 \times 10^{10}$	$18{\cdot}9 \times 10^{13}$	0·0226	$1{\cdot}26 \times 10^{15}$

Table IV.—Platinum.

$v = 2{\cdot}68 \times 10^{10}$ cm./sec.

ϕ (radians).	t_m (cms.).	$\phi/\sqrt{t_m}$.
0·24	0·000070	29
0·32	0·00014	27
0·41	0·00019	29
1·20	0·00193	27

Table IV gives the results of similar experiments with platinum. The agreement here is also satisfactory, though hardly so good as for aluminium. There is, however, in the case of platinum a source of error which does not occur in the case of aluminium. As will be seen from the table, it is necessary to use very thin leaf for the experiments on the scattering of the rays by platinum. The thickness of this can only be ascertained by weighing a known and considerable area of it. This gives the mean thickness of the leaf, but in leaf of this sort variations in thickness may occur from point to point; and thus it is quite possible that in any particular experiment the thickness of the small area (about $\frac{1}{10}$th square centimetre) actually traversed by the rays departed from the mean thickness given in Column 2.

Table III deals with the question of the variation in the magnitude of the scattering with the velocity of the rays. In performing the experiments the stop was kept the same throughout, and the thickness of aluminium required to cut down the radiation through the given stop to one-half was measured for different magnetic fields. The first column of Table III gives the product of the magnitude of the field into the radius of curvature of the path. We have then

$$\mathrm{HR} = \frac{m}{e} v$$

where m, v, and e are the mass, velocity and charge of the β-particle. Taking account of the change in mass of the β-corpuscle with velocity we have

$$\frac{e}{m} = \frac{e}{m_0}\left\{1 - \frac{v^2}{\mathrm{V}^2}\right\}^{\frac{1}{2}}$$

where V is the velocity of light. Assuming Bucherer's* value, $1{\cdot}76 \times 10^7$, for e/m_0, these equations enable us to calculate v. The product of the first two columns is equal to mv^2/e, the values of which are given in the third column of the table. The fourth column gives the corresponding values of t_m, the thickness of aluminium necessary to cut down the radiation through the fixed aperture to one-half. According to either of the equations A or B, the quotient $mv^2/e\sqrt{t_m}$ should be constant. The values of this quantity are given in the last column of the table, and it will be seen that there is again a satisfactory agreement between the theory and the experimental results. Though the actual range of velocities is not great, yet owing to the rapid change in mass with change of velocity which occurs for these fast moving particles, the value of mv^2, which is what we are really concerned with, increases more than threefold

* Bucherer, 'Ann. d. Phys.,' 1909, vol. 28, p. 524.

in the course of the series. This is a sufficient proportional increase to afford a reasonable test of the theory.

Having now obtained satisfactory evidence that, in the three directions where it lends itself to experimental verification, the theory of Prof. Thomson does express the experimental facts concerning the scattering of homogeneous β-rays, we may now proceed to apply it to the determination of the number of electrons contained in an atom of the various elements concerned.

On the Number of Electrons in an Atom.

We are presented in equations (A) and (B) with two different solutions of the problem, based on two different assumptions; the first, on the assumption that the positive electricity is uniformly distributed through the atom, the second, on the assumption that it is segregated into small units, comparable in size with the negative electron. Since both expressions lead to the same experimental laws, the present experiments, taken by themselves, do not enable us to decide between them. I have, therefore, calculated the value of N_0, the number of corpuscles in the atom, from both expressions, and both values are given in Table V. We shall see, however, when we come to consider the results that, in the light of other experiments, we must regard the first hypothesis as giving the true result.

From equation (A) we have

$$\frac{\phi_m}{\sqrt{t}} = \frac{e^2}{mv^2}\left\{\frac{384}{25}N_0+\frac{\pi^2}{16}N_0^2\right\}^{\frac{1}{2}}\sqrt{N\pi},$$

where ϕ_m is the average deflection produced in passing through a thickness t.

If ϕ is a given angle (fixed by the stop employed), and t_m the corresponding thickness of material required to cut down the radiation through the stop to one-half, we can easily show, from equation (a), that

$$\phi/\sqrt{t_m} = \phi_m/\sqrt{t}\times\sqrt{\log_e 2}.$$

Substituting in equation (A) we get

$$\phi/\sqrt{t_m} = \frac{e^2}{mv^2}\sqrt{\log_e 2}\left\{\frac{384}{25}N_0+\frac{\pi^2}{16}N_0^2\right\}^{\frac{1}{2}}\sqrt{N\pi},$$

where N_0 is the number of corpuscles in the atom, N the number of atoms per unit volume, e the charge on the β-particle in electrostatic units.

The value assumed for e is that given by Rutherford* in his paper on the charge on the α particle, namely $4{\cdot}65\times10^{-10}$ E.S. units; and N has been calculated for each element employed from the atomic weight and the density, the mass of the hydrogen atom being taken as $1{\cdot}61\times10^{-24}$ grammes. The

* Rutherford and Geiger, 'Roy. Soc. Proc.,' 1908, A, vol. 81, p. 162.

values of $\phi/\sqrt{t_m}$ and the mass and velocity of the incident β-particles are known from the present experiments.

The value of HR for the field used in the following experiments was 3420 (Gauss centimetres), this being the field which gave the maximum β-ray effect in the ionisation chamber. The corresponding values of v and m are $2{\cdot}681 \times 10^{10}$ cm./sec. and $1{\cdot}96 \times 10^{-27}$ grammes respectively.

Substituting these values, we get from (A)

$$\phi/\sqrt{t_m} = 1{\cdot}84 \times 10^{-13}\,\{15{\cdot}3\,\mathrm{N}_0 + 0{\cdot}62\,\mathrm{N}_0^2\}^{\frac{1}{2}}\sqrt{\pi\mathrm{N}},$$

and from (B)

$$\phi/\sqrt{t_m} = 1{\cdot}84 \times 10^{-13}\,[15{\cdot}3\,\mathrm{N}_0\,\{2-(1-\pi/8)\,\sigma^{\frac{1}{3}}\}]^{\frac{1}{2}}\sqrt{\pi\mathrm{N}}.$$

The value of σ, the ratio of the volume of the positive electrification to the volume of the atom, on the assumption that the positive electricity is aggregated into small discrete bundles, is not known. If we assume that the bundles are similar in size to the negative electrons, σ will be negligibly small. In any case it must be less than unity. In calculating the results given in Column 5 of Table V, we have assumed that it may be neglected. If, however, it has the maximum value 1, the results in this column would merely have to be multiplied by the factor 1·43.

Table V.

Element.	Atomic weight.	$\phi/\sqrt{t_m}$.	N_0.		N_0/Atomic weight.	
			A.	B.	A.	B.
Carbon*	12	2·0	40	44	3·32	3·7
Aluminium	27	4·25	83	156	3·07	5·8
Copper	63·2	10·0	181	765	2·87	12·0
Silver	108	15·4	320	2080	2·96	19·2
Platinum	194	29·0	605	6500	3·12	33·5

* The values for carbon were calculated from results obtained in scattering by thin films of caoutchouc. This substance contains 90 per cent. carbon, the remaining 10 per cent. being hydrogen. It was assumed in the calculations that the number of corpuscles in an atom of hydrogen had the same ratio to the atomic weight as in the carbon atom. The weight of hydrogen present is very small and cannot seriously affect the result.

Table V contains the results obtained for five different elements. The first column contains the name of the element, the second its atomic weight, the third the observed value of the constant $\phi/\sqrt{t_m}$. In each case the value given is the mean of several concordant results, using stops of different sizes, and therefore different values of ϕ. The fourth and fifth columns give

the values of N_0, the number of electrons in the atom, on the two different assumptions (A) and (B). Columns 6 and 7 give the ratio of the number of electrons in the atom to the atomic weight, on the two assumptions.

It will at once be noticed that the ratio of the number of electrons in the atom to the atomic weight, calculated on the hypothesis (A) that the positive electricity is distributed uniformly through the atom, is remarkably constant; while, on the other hand, the results obtained on the contrary hypothesis (B), that the positive electricity is in an electronic condition in the atom, lead to a very rapid increase in this ratio with increasing atomic weight; the ratio for platinum being nearly nine times that for carbon. This holds good whatever value is assumed for σ.

From other considerations, and in particular from a consideration of the scattering of the Röntgen rays by gases, it seems very probable that the number of corpuscles in an atom is, at any rate, very nearly proportional to the atomic weight. It has been shown* that for gases in which the homogeneous secondary Röntgen radiation is not excited, the amount of primary radiation scattered by the gas is simply proportional to the mass of gas present, and practically independent of its chemical nature. This leads us at once to the conclusion that the number of electrons present in the atom is simply proportional to the atomic weight. It is quite incompatible with such a large increase in the ratio of N_0 to the atomic weight as is given by equation (B). We must conclude, therefore, that the positive electricity in the atom is not in a state comparable to that of the electron, but that it occupies such comparatively large volumes as to be capable of being considered as uniformly distributed over the whole atom.

Taking the values of the ratio of N_0 to the atomic weight given by equation (A), therefore, we see that the mean value of this ratio is very nearly 3·0, and that, with the exception of the results for carbon, which are open to a little uncertainty, the maximum variations from this value do not amount to more than about 4 per cent. This is not greater than the possible errors of the experiment, and there is no indication of any systematic variation in the ratio either with density or with atomic weight. We conclude, therefore, that the number of electrons in an atom is equal to three times the atomic weight.

Any error in the measurement of the magnetic field would of course affect all the results to very nearly the same extent. As the velocity varies very slowly with the magnetic field, any error introduced from this cause would be proportionately equal to the error in the measurement of the field. The magnetic field was measured with a Grassot fluxmeter which had been

* J. A. Crowther, 'Phil. Mag.,' 1907, vol. 14, p. 653.

tested immediately before the experiments. The readings obtained with this instrument were very constant, and it is hardly possible that any serious error should have crept in from this source.

On the Absorption of Homogeneous β-rays.

If in our experiments we make $\phi = \pi/2$, that is to say, if we include the whole of the rays which emerge at the further side of the absorbing sheet, we return to the conditions of an absorption experiment. As the thickness t_m necessary to cut down the emergent radiation to one-half varies as the square of the emergent angle, it is evident that much greater thicknesses of material will be required for such experiments than is the case where the angle ϕ is kept small. The theory of the scattering of the rays, with which we have been dealing, only strictly applies to very thin sheets of material, and we should expect discrepancies to arise from the mere increase in the thickness of the absorbing sheet. In addition to this, we have also the complications introduced by the gradual change in velocity of the rays during their passage through such sheets. However, it was thought that experiments on the absorption of a beam of homogeneous β-rays, from this point of view, might prove interesting, and might help to elucidate the results obtained by Wilson.

The vacuum chamber S was removed, the exit tube D being closed with a thin aluminium window (0·002 cm. thick). The absorption due to this window could be obtained by extrapolation from the experimental curve. It was, however, negligible. The ionisation chamber T was replaced by one of hemispherical shape, again in order to ensure that all the rays emerging from D, at whatever angle, should have the same length of path in the ionisation chamber. By means of metal slides, sheets of absorbing substance of different thicknesses could be inserted in the path of the rays at D. The methods of measurement were otherwise the same as those described above.

Experiments were made with aluminium and platinum. The results obtained are given in Table VI, and the values of I/I_0 for aluminium are plotted against t the thickness in fig. 5 On comparing this curve with the curve for the scattering by aluminium, it will be at once perceived that both have the same general characteristics. The intensity of the transmitted radiation at first decreases very slowly as the thickness of the absorbing sheet is increased, then much more rapidly, and finally more slowly again as the curve approaches the axis of t. Owing to the small amount of radiation transmitted it was impossible to determine the exact shape of this last portion of the curve. It did not differ appreciably from an exponential curve.

Table VI.

Thickness t (cms.).	I/I_0.	$\log_{10} I/I_0$.	$t \log_e (1-I/I_0)$.
	Aluminium.		
0·002	1·00	—	—
0·004	0·996	—	—
0·012	0·95	$\bar{1}$·98	−0·037
0·033	0·77	$\bar{1}$·99	−0·048
0·049	0·62	$\bar{1}$·79	−0·046
0·073	0·29	$\bar{1}$·46	−0·023
0·110	0·12	$\bar{1}$·08	−0·016
	Platinum.		
0·00	1·00	—	—
0·00107	0·71	$\bar{1}$·85	−0·00134
0·00220	0·53	$\bar{1}$·72	−0·00168
0·00327	0·38	$\bar{1}$·58	−0·00159
0·00556	0·23	$\bar{1}$·36	−0·00140

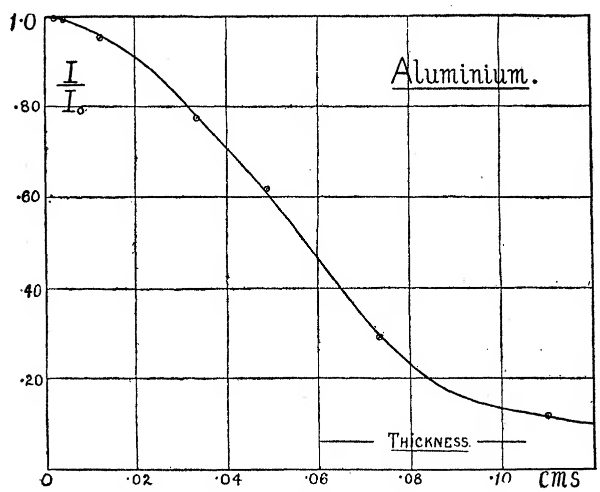

Fig. 5.

Turning again to the table, we find that the values of $t \log_e (1-I/I_0)$ given in the last column of the table remain appreciably constant until a thickness of about 0·05 cm. is reached; after this point they begin to show a rapid decrease. Taking the first portion of the curve then, and calculating the corresponding value of $\phi/\sqrt{t_m}$ as before, we obtain the value 4·5 given below the dotted line in Table II. As will be seen from that table, this value agrees well with the values obtained from the experiments on the scattering. The first stage in the absorption of such a homogeneous pencil of rays

as we have been considering consists, therefore, in the scattering or diffusion of the beam according to the laws we have already considered.

We may now attempt to give some explanation of the "linear" law propounded by W. Wilson.* The curves given in his paper for aluminium, which is the only element for which he gives the figures, differ from the one drawn in fig. 5 principally in not showing the initial gradual decrease and the first point of inflection. In general, they do show the gradual bending round of the final portion along the axis of t, though Wilson himself seems inclined to ascribe this result to experimental imperfections. Now in Wilson's experiments the β-rays fall upon his absorbing sheets in a fairly narrow pencil; they should, therefore, exhibit the effects of scattering as described above. On the other hand, as the beam was not deflected in a vacuum, but was passing through air along its whole course, it was already partially scattered before it fell upon the absorbing sheets. Thus the portion of the scattering which corresponds to the nearly horizontal initial portion of the curve had taken place before his measurements commenced. It will be seen that the middle portion of the curve in fig. 5 is approximately a straight line.

Turning now to the case of platinum (fig. 6), the curve presents a somewhat different appearance. Here again, however, if we consider only the first

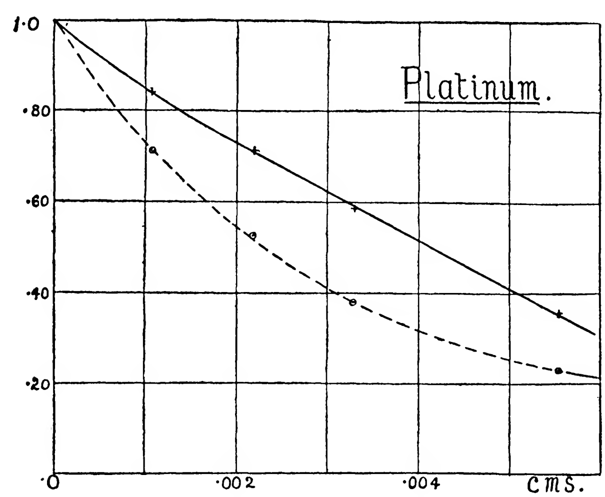

FIG. 6.

portion of the curve, that is to say, the portion corresponding to thicknesses less than 0·001 cm., the results connect themselves, as in the case of aluminium, with the experiments on scattering described above, the value of

* W. Wilson, 'Roy. Soc. Proc.,' 1909, A, vol. 82, p. 612.

$\phi/\sqrt{t_m}$ obtained from this portion being 27, as compared with the value 29 from the previous experiments.

After passing through 0·001 cm. of platinum, however, the absorption of the rays becomes exponential, as is shown by the unbroken curve in fig. 6, which gives the values of $\log_{10}(I/I_0)$ plotted against the corresponding values of t. Thus the absorption of homogeneous β-rays, after scattering in a thickness of only 0·001 cm. of platinum, follows the law which has always been found for the absorption of the rays from a single radioactive substance.

Table VII.—Absorption of the homogeneous β-rays by Aluminium after transmission through 0·001 cm. of Platinum.

t (cms.).	I/I_0.	$\log_{10} I/I_0$.
0·0	1·00	—
0·020	0·70	$\bar{1}$·84
0·048	0·41	$\bar{1}$·61
0·061	0·31	$\bar{1}$·49
0·081	0·22	$\bar{1}$·34
0·110	0·12	$\bar{1}$·08

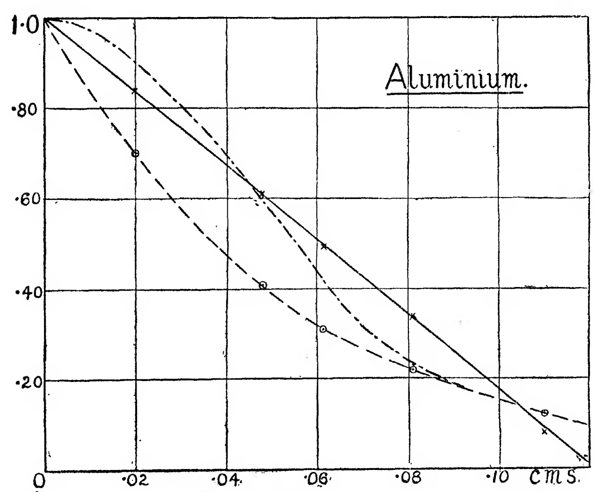

Fig. 7.

As a further test of this point, I have measured the absorption by aluminium of a beam of homogeneous β-rays which had first traversed a thin sheet of platinum. The results are given in Table VII, and graphically in fig. 7, where the full line gives the values of $\log_{10}(I/I_0)$ plotted against the thickness t. It will be seen that this is a straight line within the limits of experimental error. Thus the absorption in aluminium of homogeneous β-rays after previous scattering in 0·001 cm. of platinum follows an

exponential law. The lower dotted curve on the same diagram gives the actual values of I/I_0, while for sake of comparison the absorption curve of fig. 5 is also reproduced on the same scale. It will be seen that these two curves eventually coincide. It seems probable, therefore, that the apparent difference in the results obtained for platinum and aluminium is one of degree only, and that, as suggested above, the homogeneous rays after scattering in aluminium also follow an exponential law of absorption. We are thus led to the conclusion that, while the scattering of a parallel pencil of homogeneous β-rays, and hence also the first stages in the absorption of a parallel pencil, takes place according to the laws considered in the first part of this paper, the absorption of the completely scattered beam follows an exponential law.

Wilson, who regarded his "linear" curves as giving the true law of absorption for homogeneous rays, has attempted to show that an exponential law of absorption is due to a peculiar distribution of velocities in the rays employed. It seems much more probable, however, from the above results that the exponential is the true absorption law for completely scattered β-radiation of uniform velocity, whether it emerges from a layer of radioactive material, or has been originally separated out by a magnetic field, and afterwards scattered in some absorbing substance.

In a recent paper Baeyer and Hahn* have published some excellent photographs of the magnetic spectra of various radioactive substances and their mixtures, from which it seems clear that the velocity of the β-rays emitted by any one radioactive substance, such as radium E, or the different thorium products, is, if not absolutely constant, at any rate confined within very narrow limits, and H. W. Schmidt† announces a similar result for the β-rays from uranium X, though Gray (in a paper recently read before this Society, the details of which are not to hand at the time of writing) has failed to observe any such effect.

It may be pointed out that the absorption coefficients obtained for platinum and aluminium after the scattering of the rays in platinum foil are in good agreement with the values to be expected from the velocity of the incident homogeneous rays. The velocity of the homogeneous beam used in these experiments was $2{\cdot}68 \times 10^{10}$ cm./sec., and the values obtained for λ/ρ (the coefficient of absorption divided by the density) for platinum and aluminium after the scattering of the rays were 11·9 and 6·9 respectively. The rays from uranium X, according to Schmidt,‡ have

* v. Baeyer and Hahn, 'Phys. Zeit.,' 1910, vol. 11, p. 488.

† H. W. Schmidt, 'Phys. Zeit.,' 1910, vol. 11, p. 265.

‡ H. W. Schmidt, 'Le Radium,' 1909, vol. 6, p. 5.

the velocity $2{\cdot}76 \times 10^{10}$ cm./sec., and the corresponding values of λ/ρ for these rays are 9·4 and 5·3.* Remembering that the coefficient of absorption is probably proportional to the inverse fourth power of the velocity of the rays, it will be seen that the agreement between the two sets of figures obtained in these very different ways is very satisfactory.

As to the exact nature of the processes involved in the exponential absorption of the diffuse rays it would, perhaps, be premature to decide. Makower,† who measured the actual numbers of β-particles from radium emanation transmitted by glass cylinders of various thicknesses, obtained results for the absorption of glass very similar to those obtained by ionisation methods. It would seem, therefore, that the absorption involves the actual stoppage of a certain proportion of the β-particles in each layer of absorbing material, the number stopped in a given small thickness being proportional to the total number actually present. Further experiments on this point might prove interesting.

Summary.

I. The scattering of a homogeneous pencil of β-rays has been measured for various substances and for rays of different velocity. It has been shown to obey the following laws:—

(i) For rays of given velocity the intensity I of the radiation contained within a given cone may be expressed by the equation—

$$I/I_0 = 1 - e^{-k/t},$$

where t is the thickness of material passed through by the rays, and k is a constant, depending upon the angle of the cone.

(ii) For rays of given velocity the most probable angle of emergence is proportional to the square root of the thickness of material traversed by the rays.

(iii) For rays of different velocity the probable angle of emergence divided by the square root of the thickness traversed is inversely proportional to the product of the mass of the β-particle and the square of its velocity.

II. From equations given by Prof. Sir J. J. Thomson, the number of electrons contained in atoms of different elements is calculated. It is found—

(i) That the ratio of the number of electrons in an atom to the atomic weight is constant, the value of the ratio being very nearly 3·0 for all the elements examined.

* J. A. Crowther, 'Phil. Mag.,' 1906, vol. 12, p. 379.
† Makower, 'Phil. Mag.,' January, 1909.

(ii) That the positive electricity within the atom is not in an electronic condition, but is distributed fairly uniformly through the atom.

III. Experiments are described on the absorption of the homogeneous β-rays. It is shown that the first stage in the absorption of a pencil of homogeneous β-rays consists in the scattering of the rays. The absorption of a completely scattered beam of homogeneous β-rays is shown to take place according to an exponential law.

In conclusion, I have much pleasure in once more recording my best thanks to Prof. Sir J. J. Thomson for much inspiration and advice during the course of these experiments.

Aerial Plane Waves of Finite Amplitude.

By Lord Rayleigh, O.M., F.R.S.

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Waves of Finite Amplitude without Dissipation.

In the investigations which follow, we are concerned with the motion of an elastic fluid in one dimension, say, parallel to x. It is implied not only that there are no component velocities perpendicular to x, but that the motion is the same in any perpendicular plane, so that it is a function of x and of the time (t) only. If u be the velocity at any point x, p the pressure, ρ the density, X an impressed force, the dynamical equation for an inviscid fluid is

$$\frac{du}{dt}+u\frac{du}{dx} = \mathrm{X}-\frac{1}{\rho}\frac{dp}{dx}. \tag{1}$$

At the same time the "equation of continuity" takes the form

$$\frac{d\rho}{dt}+\frac{d(\rho u)}{dx} = 0. \tag{2}$$

The first step, and it was a very important one, in the treatment of waves of finite amplitude is due to Poisson.* Under the assumption of Boyle's law, $p = a^2\rho$, he proved that for waves travelling in one direction (positive) the circumstances of the propagation are expressed by

$$u = f\{x-(a+u)t\}, \tag{3}$$

* "Mémoire sur la Théorie du Son," 'Journ. de l'École Polytechnique, 1808, vol. 7 p. 319.